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CONJUGATE PROBLEM OF HEAT EXCHANGE WITH RADIANT HEAT
EXCHANGE NEAR PLATE SURFACE

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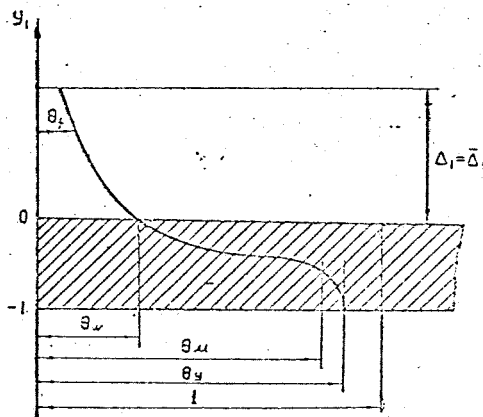
CONJUGATE PROBLEM OF HEAT EXCHANGE WITH RADIANT HEAT EXCHANGE NEAR PLATE SURFACE

L. I. Kudryashev, N. L. Men'shikh

ABSTRACT. The conjugate problem of heat exchange with radiant heat exchange near the surface of a plate is broken down into two different problems. The first is heating of a plate that is adiabatic on one side during radiation from the opposite surface in an absolutely transparent gas. The second is the problem where heating of the surface of a plate occurs not only as the result of convective heat exchange, but also of radiant heat exchange with the radiation of the combustion products.

Conjugate problems of the determination of the heat transfer coefficient under conditions of thermal transience, complicated by radiant heat exchange, are often encountered in thermal calculations of structural elements of flight vehicles and their engines. Problems of this type have not been treated in literature, despite their great practical importance. This can be attributed to the tremendous difficulties involved in the integration of the equation systems that solve the problem and the nonlinear boundary condition due to radiant heat exchange. This problem can be broken down into two separate problems that differ from each other substantially. The first, representing

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the problem of the heating of a plate that is adiabatic on one side with respect to radiation from the opposite surface in an absolutely transparent gas. The second is the problem where the surface of a plate is heated not only by means of convective, but also by radiative heat exchange as the result of the radiation of the combustion products. The figure shows the diagram of the problem under consideration. In accordance with this diagram the mathematical

*Numbers in the margin indicate pagination in the foreign text.

formulation of the problem is written as follows:

for $y_1 \leq 0$

$$\frac{\partial \theta_M}{\partial Fo_M} = \frac{\partial^2 \theta_M}{\partial y_1^2}, \quad (1a)$$

$$Fo_M = 0, \quad \theta_M = 1, \quad (1b)$$

$$-\left(\frac{\partial \theta_M}{\partial y_1}\right)_{y_1=0} = Bi_K \theta_W \left\{ 1 \pm St \frac{\left[1 - \left(1 - \frac{T_\infty - T_0}{T_\infty} \theta_W \right)^4 \right]}{\theta_W} \right\}, \quad (1c)$$

$$-\left(\frac{\partial \theta_M}{\partial y_1}\right)_{y_1=-1} = 0, \quad (1d)$$

for $y' \geq 0$

$$Re_\infty \left(w_{x_1} \frac{\partial w_{x_1}}{\partial x_1} + w_{y_1'} \frac{\partial w_{x_1}}{\partial y_1'} \right) = \frac{\partial^2 w_{x_1}}{\partial x_1^2} + \frac{\partial^2 w_{x_1}}{\partial y_1'^2}, \quad (2a)$$

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$$Re_\infty \left(w_{x_1} \frac{\partial w_{y_1}}{\partial x_1} + w_{y_1'} \frac{\partial w_{y_1}}{\partial y_1'} \right) = \frac{\partial^2 w_{y_1}}{\partial x_1^2} + \frac{\partial^2 w_{y_1}}{\partial y_1'^2}, \quad (2b)$$

$$\frac{\partial x_{x_1}}{\partial x_1} + \frac{\partial w_{y_1'}}{\partial y_1} = 0, \quad (2c)$$

$$\frac{\partial \theta_f}{\partial Fo_f} + Pe_\infty \left(w_{x_1} \frac{\partial \theta_f}{\partial x_1} + w_{y_1'} \frac{\partial \theta_f}{\partial y_1'} \right) = \frac{\partial^2 \theta_f}{\partial x_1^2} + \frac{\partial^2 \theta_f}{\partial y_1'^2}, \quad (2d)$$

$$Fo_f = 0, \quad \theta_f = 0, \quad (2e)$$

$$-\left(\frac{\partial \theta_f}{\partial y_1'}\right)_{y_1'=0} = Nu_K \theta_W \left\{ 1 \pm St \frac{\left[1 - \left(1 - \frac{T_\infty - T_0}{T_\infty} \theta_W \right)^4 \right]}{\theta_W} \right\}, \quad (2f)$$

$$y_1' = 0, \quad w_{x_1} = w_{y_1'} = 0, \quad (2g)$$

$$y_1' = \infty, \quad \theta_f = 0, \quad w_{x_1} = 1, \quad w_{y_1'} = 0, \quad (2h)$$

where

$$\theta_M = \frac{T_\infty - T_M}{T_\infty - T_0}, \quad \theta_f = \frac{T_\infty - T_f}{T_\infty - T_0}, \quad y_1 = \frac{y}{l}, \quad x_1 = \frac{x}{L},$$

$$y_1' = \frac{y}{L}, \quad Fo_M = \frac{a_M \tau}{l^2}, \quad Fo_f = \frac{a_f \tau}{L^2}, \quad Re_\infty = \frac{w_\infty L}{\nu_f},$$

$$Pe_{\infty} = \frac{w_{\infty} L}{a_f}, \quad Nu_k = \frac{\alpha_k L}{\lambda_f}, \quad Bi_k = \frac{\alpha_k l}{\lambda_M}, \quad St = \frac{\sigma_{\Pi} T_{\infty}^3}{\alpha_k \left(1 - \frac{T_0}{T_{\infty}}\right)},$$

$$w_{x_1} = \frac{w_x}{w_{\infty}}, \quad w_{y_1} = \frac{w_y}{w_{\infty}}, \quad \theta_W = \frac{T_{\infty} - T_W}{T_{\infty} - T_0}.$$

Here l and L are the thickness and the length, respectively, of the by-passed plate. For the absolutely transparent gas $\sigma = 10^{-8} C_t$, and for the radiating gas, $\sigma_t = E_W' E_2' C_0 \cdot 10^{-8}$, where E_W' is the effective blackness of the surface of the plate. Since

$$\theta_f|_{y_1'=0} = \theta_M|_{y_1'=0} = \theta_W(For), \quad (3)$$

then, by comparing (1c) and (2e), we arrive at a boundary condition of the fourth kind:

$$-\left(\frac{\partial \theta_f}{\partial y_1'}\right)_{y_1'=0} = \frac{\lambda_M L}{\lambda_f l} \left(\frac{\partial \theta_M}{\partial y_1}\right)_{y_1=0}. \quad (4)$$

The joint solution of systems (1) and (2) with consideration of condition (4) is accomplished in four steps: first the internal problem is solved, including (1a), (1b), (1d) and (3), and then the external, consisting of (2a), (2b), (2c), (2d), (2f), (3). With the solution for θ_M and θ_f , which still include the unknown temperature of the surface θ_W , and by satisfying conditions (4), we arrive at a differential equation that enables us to find $\theta_W = \theta_W(For)$. Finally, from the known values of θ_f and θ_W we find, on the basis of (2f), the number Nu_k that characterizes heat exchange.

1. Internal Problem

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For approximating the internal problem, L. S. Leybenson's integral relation is used [1, 2]:

$$\frac{\partial}{\partial For_M} \int_{-1}^0 \theta_M dy_1 = \left(\frac{\partial \theta_M}{\partial y_1}\right)_{y_1=0}. \quad (5)$$

Boundary conditions (1d) and (3) can be satisfied if the approximate solution for θ_M is represented in the form

$$\theta_M = \theta_W - C(2y_1 + y_1^2). \quad (6)$$

By substituting (6) into (5), we arrive at a differential equation relative to C , the integration of which produces:

$$C = \exp(-3Fo_M) \left\{ C_0 + \frac{3}{2} \int_0^{Fo_M} \frac{d\theta_W}{dFo_M} \exp(3Fo_M) dFo_M \right\}. \quad (7)$$

By substituting (7) into (6), and also considering (1b), we have $C_0 = 0$. Consequently, the approximate solution can be written:

$$\theta_M = \theta_W - \left\{ \frac{3}{2} \exp(-3Fo_M) \int_0^{Fo_M} \frac{d\theta_W}{dFo_M} \exp(3Fo_M) dFo_M \right\} (2y_1 + y_1^2). \quad (8)$$

2. External Problem

For the approximate solution of the external problem we will use the concept of the region of thermal influence Δ . By this region we mean that terminal region near the surface of the by-passed plate in which the normal velocity component is equal to zero, and on whose external boundary the longitudinal velocity component and temperature are assumed to be about equal to the velocity and temperature of the incident flow. The definition of Δ given here is similar to that of the thickness of hydrodynamic and thermal boundary layers. However, there are some important differences. Actually, if the scales of the transverse and longitudinal values are related by the expressions

$$\frac{w_{y'_1\infty}}{l_{y'_1}} = 1, \quad l_{y'_1}^2 Re_\infty = 1, \quad (9a)$$

where

$$l_{y'_1} = \frac{l_y}{L}, \quad w_{y'_1\infty} = \frac{w_{y,\infty}}{w_\infty}, \quad (9b)$$

then for the numbers $Re_\infty \rightarrow \infty$, (2a), (2b), and (2c) are converted to the familiar Prandtl's boundary layer equations, while (2d) is converted to the thermal boundary layer equation. If now the condition (9b) is retained and it is assumed in (9a) that $w_{y_1'} \approx 0$, then, regardless of the values of the Re_∞ numbers, equations (2a), (2b), (2c) and (2d) acquire the form

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$$\frac{\partial^2 w_{x_1}}{\partial y_1'^2} = 0, \quad (10a)$$

$$\frac{\partial w_{x_1}}{\partial x_1} = 0, \quad (10b)$$

$$\frac{\partial \theta_f}{\partial Fo} + Pe_\infty w_{x_1} \frac{\partial \theta_f}{\partial x_1} = \frac{\partial^2 \theta_f}{\partial x_1^2} + \frac{\partial^2 \theta_f}{\partial y_1'^2}. \quad (10c)$$

Moreover, assuming that $\frac{\partial^2 \theta_f}{\partial x_1^2}$ dissipates little in comparison with $\frac{\partial^2 \theta_f}{\partial y_1'^2}$, equation (10c) can be written

$$\frac{\partial^2 \theta_f}{\partial Fo} + Pe_\infty w_{x_1} \frac{\partial \theta_f}{\partial x_1} = \frac{\partial^2 \theta_f}{\partial y_1'^2}. \quad (10c')$$

It is clear then that the definition given herein for the range of influence is more general in comparison with the theory of a boundary layer of finite thickness, since it is not related to the condition $Re_\infty \rightarrow \infty$ when the approximate velocity and temperature fields are retained in the range of thermal influence. This is particularly important, for instance, for investigations of heat exchange during flow around a sphere in the range of small Reynolds numbers [3, 4].

Therefore the solution of the problem of heat exchange incorporating the range of influence Δ reduces to the solution of equation (10c') for the following initial and boundary conditions:

$$0 \leq x_1 \leq 1$$

Here $\eta = \frac{y_1'}{\Delta_1}$, $\Delta_1 = \frac{\Delta}{L}$.

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follows:

(12)

where $Fo'_f = \frac{Fo_f}{Fo_{fei}}$.

(12) will be of the same order of magnitude, it is necessary that

(13)

i.e., $F_{o_{fej}}$ is very small in magnitude.

Consequently, after integrating equation (10c) we may limit the discussion to simply the quasistationary period. If we look for the solution in the form of the sum

$$\theta_f = \theta_{f_1} + \theta_{f_2}, \quad (14)$$

where

$$\theta_{f_1} = \theta_{f_1}(x_1, y_1) \quad \theta_{f_2} = \theta_{f_2}(Fo_f, y_1), \quad (15)$$

then (10c) is broken down into two equations

$$Pe_\infty w_{x_1} \frac{\partial \theta_f}{\partial x_1} = \frac{\partial^2 \theta_{f_1}}{\partial y_1^2} \quad (16)$$

for boundary conditions (11b), (11c), (11d), (11e) and

$$\frac{\partial^2 \theta_f}{\partial Fo_f} = \frac{\partial^2 \theta_{f_2}}{\partial y_1^2}, \quad (17)$$

for the initial condition (11a) and boundary conditions:

$$\eta = 0, \theta_{f_2} = 0, \quad (18a) \quad \eta = 1, \theta_{f_2} = 0. \quad (18b)$$

The following integral relations correspond to equations (16) and (17):

$$\frac{\partial \Delta_1}{\partial x_1} \int_0^1 w_{x_1} \theta_{f_1} d\eta = - \frac{1}{Pe_\infty} \frac{1}{\Delta_1} \left(\frac{\partial \theta_{f_1}}{\partial \eta} \right)_{\eta=0}, \quad (19a)$$

$$\frac{\partial}{\partial Fo_f} \int_0^1 \theta_{f_2} d\eta = \frac{1}{\Delta_1^2} \left[\left(\frac{\partial \theta_{f_2}}{\partial \eta} \right)_{\eta=1} - \left(\frac{\partial \theta_{f_2}}{\partial \eta} \right)_{\eta=0} \right]. \quad (19b)$$

If we confine ourselves to the fourth approximation, then the boundary conditions (11b), (11c), (11d) and (11e) will be satisfied by the polynomials

$$w_{x_1} = 2\eta - 2\eta^3 + \eta^4, \quad (20a)$$

$$\theta_{f_1} = \theta_w (1 - 2\eta + 2\eta^3 - \eta^4). \quad (20b)$$

By substituting (20) into (19a), we arrive at a differential equation relative to Δ , integration of which yields:

$$\Delta_1 = 5,83 \sqrt{\frac{x_1}{Pe_\infty}}. \quad (21)$$

The average value is

$$\bar{\Delta}_1 = \frac{3,89}{\sqrt{Pe_\infty}}. \quad (22)$$

Assuming Δ_1 to be unknown, we examine integral relation (19b). By multiplying both sides of (19b) by dx_1 and integrating within the boundaries of 0 to 1, we obtain

$$\frac{\partial}{\partial Fo_f} \int_0^1 \theta_{f_2} d\eta = \frac{1}{\Delta_1^2} \left[\left(\frac{\partial \theta_{f_2}}{\partial \eta} \right)_{\eta=1} - \left(\frac{\partial \theta_{f_2}}{\partial \eta} \right)_{\eta=0} \right], \quad (23)$$

where

$$\Delta_1^2 = \int_0^1 \Delta_1^2 dx_1 = \frac{5,83^2}{2} \cdot \frac{1}{Pe_\infty} = 1,12 (\bar{\Delta}_1^2). \quad (24)$$

Boundary conditions (18) are satisfied by the polynomial

$$\theta_{f_2} = C(\eta - \eta^2). \quad (25)$$

Substituting (25) into (23) and integrating, and also recalling equation (22) and (24), we obtain

$$C = C_0 \exp\left(-\frac{6}{17} Pe_\infty Fo_f\right). \quad (26)$$

After substituting (20b) and (25) into (14), considering equation (26), we will have

$$\theta_f = \theta_w(1 - 2\eta + 2\eta^3 - \eta^4) + C_0(\eta - \eta^2) \exp\left(-\frac{6}{17} Pe_\infty Fo_f\right). \quad (27)$$

When $Fo_f = 0$, $\theta_f = 0$, $\theta_w = 1$. Consequently

$$0 = (1 - 2\eta + 2\eta^3 - \eta^4) + C_0(\eta - \eta^2).$$

We find the arbitrary integration constant C_0 from the condition of the minimum functional

$$\delta \int_0^1 [(1 - 2\eta + 2\eta^3 - \eta^4) + C_0(\eta - \eta^2)]^2 dy_1 = 0, \quad (28)$$

hence

$$C_0 = - \frac{\int_0^1 (1 - 2\eta + 2\eta^3 - \eta^4)(\eta - \eta^2) d\eta}{\int_0^1 (\eta - \eta^2)^2 d\eta} = -\frac{9}{7}. \quad (29)$$

By substituting (29) into (27), we finally obtain the approximate solution of the external problem:

$$\theta_f = \theta_w(1 - 2\eta + 2\eta^3 - \eta^4) - \frac{9}{7}(\eta - \eta^2) \exp\left(-\frac{6}{17} \text{Pe}_\infty \text{Fo}_f\right). \quad (30)$$

Equation (4) can be written in the form

$$\frac{1}{\Delta_1} \left(\frac{\partial \theta_f}{\partial \eta} \right)_{\eta=0} = \frac{\lambda_M L}{\lambda_f l} \left(\frac{\partial \theta_M}{\partial y_1} \right)_{y_1=0}. \quad (31)$$

By multiplying both sides of this equation by dx and integrating from 0 to 1, we will obtain

$$\frac{1,335}{\Delta_1} \left(\frac{\partial \theta_f}{\partial \eta} \right)_{\eta=0} = \frac{\lambda_M L}{\lambda_f l} \left(\frac{\partial \theta_M}{\partial y_1} \right)_{y_1=0}. \quad (32)$$

Now, by substituting (8) and (30) into (32), we arrive at the following integro-differential equation relative to the surface temperature θ_w : /83.

$$\begin{aligned} \theta_w + 0,643 \exp\left(-\frac{6}{17} \text{Pe}_\infty \text{Fo}_f\right) &= 1,123 \frac{\lambda_M L}{\lambda_f l} \exp(-3\text{Fo}_M) \times \\ &\times \int_0^{\text{Fo}_M} \frac{d\theta_w}{d\text{Fo}_M} \exp(3\text{Fo}_M) d\text{Fo}_M \end{aligned} \quad (33)$$

or

$$\frac{d\theta_W}{dFo_M} + A\theta_W + B \exp\left(-\frac{6}{17} D Pe_\infty Fo_M\right) = 0, \quad (34)$$

where

$$A = \frac{3}{1 - 1,123 \frac{\lambda_M L}{\lambda_f l} \bar{\Delta}_1}, \quad B = \frac{1,93 - 0,227 D Pe_\infty}{1 - 1,123 \frac{\lambda_M L}{\lambda_f l} \bar{\Delta}_1}, \quad D = \frac{a_f l^2}{a_M L^2}.$$

The integral of equation (34) for the initial condition $Fo_M = 0$, $\theta_W = 1$, will be

$$\theta_W = (1 - f) \exp(-A Fo_M), \quad (35)$$

where

$$f = \frac{B}{A - \frac{6}{17} D Re_\infty} \left[1 - \exp\left(A - \frac{6}{17} D Pe_\infty\right) Fo_M \right]. \quad (36)$$

The heat transfer coefficient is found from (2b). This yields

$$Nu_k = - \frac{1}{\Delta\theta_W} \left(\frac{\partial\theta_f}{\partial\eta} \right)_{\eta=0} \frac{\theta_W}{\theta_W \pm st \left[1 - \left(1 - \frac{T_\infty - T_0}{T_\infty} \theta_W \right)^4 \right]}. \quad (37)$$

Considering (21), (30) and (35), and then taking the average with respect to x_1 , we find:

$$\begin{aligned} \bar{Nu}_k &= 0,687 Re_\infty^{0,5} \left[1 + \frac{9}{14(1-f) \exp(-A Fo_M)} \exp\left(-\frac{6}{17} Pe_\infty Fo_f\right) \right] \times \\ &\times \frac{(1-f) \exp(-A Fo_M)}{(1-f) \exp(-A Fo_M) \pm st \left\{ 1 - \left[1 - \frac{T_\infty - T_0}{T_\infty} (1-f) \exp(-A Fo_M) \right] \right\}}. \end{aligned} \quad (38)$$

Equation (38) takes into account in explicit form the effect of the internal and external thermal transience as well as the effect of radiant heat exchange

on the coefficient of convective heat exchange. The effect of the latter is particularly important for the thermal calculation of combustion chambers of engines for flight vehicles, where the contribution of radiant heat transfer is considerable.

In the partial case, if $Fo_M \rightarrow \infty$, then

$$\overline{Nu}_k = 0,687 Re_\infty^{0.5} Pr^{1/2}. \quad (39)$$

Equation (39) coincides closely with the exact Polhausen solution for multiatomic gases. Actually, if (39) is written in the form

$$\overline{Nu}_k = (0,687 Pr^{1/6}) Re_\infty^{0.5} Pr^{1/3} \quad (40)$$

and compared with the exact Polhausen solution [5]

$$\overline{Nu}_k = 0,664 Re_\infty^{0.5} Pr^{1/3}, \quad (41)$$

then the ratio is

$$\frac{\overline{Nu}_{k(40)}}{\overline{Nu}_{k(41)}} = 1,035 Pr^{1/6},$$

i.e., divergence of the exact and approximate solutions for a gas medium is, for all practical purposes, small.

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